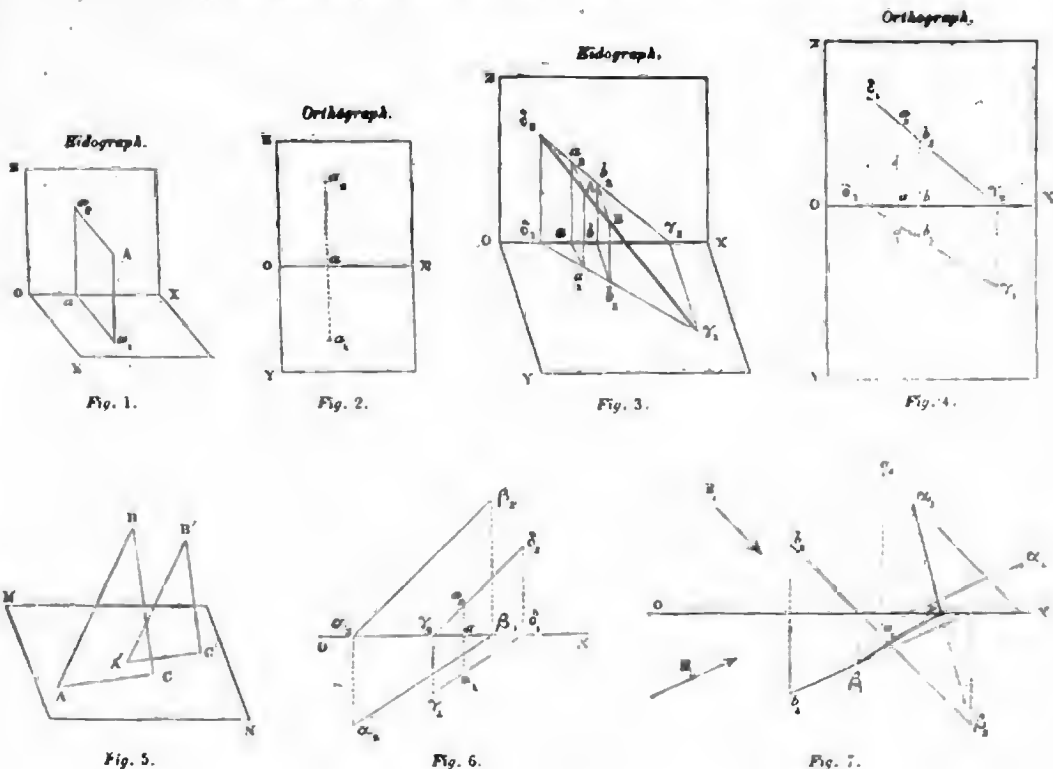


ILLUSTRATIONS OF ARCHITECTURAL SHADOWS.



plane  $XY$  (xi. 38), and hence (xi. 6)  $a_n$  is parallel to  $Aa_1$ . Similarly it is proved that  $a_1A$  is parallel to  $a_n$ ; and hence  $Aa_1a_n$  is a parallelogram. Also it has one right angle  $a_1aa_n$ , and is therefore a rectangle (i. 29 and 34).

Let us now conceive the lines  $Aa_1$ ,  $Aa_n$  to be removed, leaving the lines  $a_1a$ ,  $a_n$  in their respective positions in the planes  $XY$ ,  $XZ$ . Then let us conceive the plane  $XY$  to be revolved about  $OX$  till it comes into coincidence with the plane  $XZ$  produced. The result will be the *orthograph*. Then since  $Xa_1a$  is a right angle, and the relation of  $Xa$  to  $a_n$  is not altered by the revolution about  $Xa$ , it will be a right angle in the *orthograph*; and  $a_1aa_n$  becomes a single straight line perpendicular to  $OX$ .

In this it will be seen at once, that  $a_n$  is the height of the point  $A$  of the *eidograph* above the plane  $XY$ , and that  $a_1a$  is the distance of the same point  $A$  before the elevation  $XZ$ . And if we take away the frame-work of the planes  $XY$ ,  $XZ$ , we shall have the ordinary "descriptive representation" of the point  $A$  of the *eidograph*.

Again, if there be a second point  $B$  in the *eidograph*, its "descriptive representation" will be  $b_1b_n$ , as in the figured *orthograph*.

Also, if  $AB$  be a given line in the *eidograph*, the projections  $a_1b_1$ ,  $a_nb_n$  constitute its descriptive representations in the *orthograph*. Figs 3 and 4.

II. Let us now conceive the line  $AB$  produced both ways (if necessary) to pierce the plan and elevation planes in  $\gamma_1$  and  $\gamma_n$ . These are called the *traces* of the line  $AB$ . Then two converse problems arise, which it will be necessary to specify.

**PROBLEM A.**—Given the projections of the two points in a line, to find the traces of that line.

**ANALYSIS.**—Let the *eidographic* plane  $AB\delta_1\mu$  be produced and cut  $XY$ ,  $XZ$ , in  $\gamma_1\delta_1$ ,  $\delta_1\delta_n$  and the plane  $AB\delta_1\mu$  to cut them in  $\gamma_1\gamma_n$ ,  $\gamma_n\delta_n$ . Then, since both planes pass through  $AB$ , their intersection is the line  $AB$  produced to meet the planes  $XY$ ,  $XZ$ ; or  $\gamma_1$ ,  $\delta_n$  will be the traces of the line  $AB$ .

Now since by the definition of *orthographic* projection,  $Aa_1$  and  $Bb_1$  are perpendicular to

$XY$ , the plane  $\gamma_1\delta_1\delta_n$  is perpendicular to  $XY$  (xi. 18). Wherefore, since the two planes  $\gamma_1\delta_1\delta_n$  and  $XZ$  are perpendicular to  $XY$ , their intersection  $\delta_1\delta_n$  is also perpendicular to  $XY$  (xi. 19); and hence (xi. 4)  $\delta_1\delta_n$  is perpendicular to  $OX$ . But  $a_1$ ,  $b_1$  are given, and hence the intersection  $\delta_1$  of  $a_1b_1$  with  $OX$  is given; and therefore the vertical trace of the line  $AB$  is in the given perpendicular from  $\delta_1$ . Moreover, the vertical trace of  $AB$  is in the given vertical projection  $a_nb_n$  of the line  $AB$ : Whence it is their intersection.

In the same way the horizontal trace is shown to be the intersection of  $a_1b_1$  with  $\gamma_1\gamma_n$ ; and the following simple construction results:—

**SYNTHESIS.**—Let  $a_1b_1$ ,  $a_nb_n$  be the horizontal and vertical projections of the line  $AB$  upon the *orthograph*. Produce them to meet the ground line  $OX$  in  $\delta_1$  and  $\gamma_1$  respectively. Draw the perpendicular  $\gamma_1\gamma_n$ ,  $\delta_1\delta_n$  to meet the projections  $a_1b_1$ ,  $a_nb_n$  (produced if necessary) in  $\gamma_1$ ,  $\delta_n$ . There will be the traces required.

The demonstration is obvious from the analysis and need not be put down here.

**PROB. B.**—Given the traces of a line to find its projections.

The following construction results from a slight variation of the preceding analysis.

**SYNTHESIS.**—Let  $\gamma_1$ ,  $\delta_n$  be the traces on the *orthograph* of the given line. Draw  $\gamma_1\gamma_n$ ,  $\delta_1\delta_n$  perpendicular to the ground line  $OX$ ; and join  $\gamma_1\delta_1$ ,  $\gamma_n\delta_n$ . These are the projections of the entire of the visible part of the line; that is, of the part which lies in the "region of space" enclosed by the two planes which are opposite to the spectator.

III. The projections of parallel lines are themselves parallel, whatever be the plane of projection, and whatever be the direction of the projectors.

Let  $MN$  be the plane of projection,  $AB$ ,  $A'B'$  two parallel lines meeting  $MN$  in  $A$ ,  $A'$ . Take any points,  $B$ ,  $B'$  in these lines, and draw  $BC$ ,  $B'C'$  parallel to the pattern-projector, meeting  $MN$  in  $C$ ,  $C'$ .

Then, since  $BC$ ,  $B'C'$  are parallel to the pattern-projector, they are parallel to one another (xi. 9). Wherefore the two lines,  $AB$ ,  $BC$ , are respectively parallel to the two

lines  $A'B'$ ,  $B'C'$ , and, therefore, the planes  $ABC$ ,  $A'B'C'$  are parallel (xi. 15).

Again, since the parallel planes  $ABC$ ,  $A'B'C'$  are cut by the plane  $MN$ , their sections,  $AC$ ,  $A'C'$ , with it are parallel (xi. 16); and these are the projections of  $AB$ ,  $A'B'$  upon the plane  $MN$  by projectors parallel to a given pattern-projector. Whence, &c.

The proposition as here enunciated is somewhat more general than we require just now; but as the demonstration is precisely similar to that more limited case of *orthographic* projection, I have put it down in the general form.

IV. It follows from the preceding investigation, that if the projections of two lines upon both the horizontal and vertical planes be parallel, they are the projections of parallel lines. Or, in reference to the annexed figure, where  $a_1b_1$ ,  $a_nb_n$  are the projections of a line continued through the whole visible region, and  $\gamma_1\delta_1$ ,  $\gamma_n\delta_n$  parallel respectively to the former projections are the projections of another line, the two lines projected will be parallel. Fig. 6.

It also follows, that if the projections  $a_1$ ,  $a_n$  of a point be given, and likewise those of a line  $a_1b_1$ ,  $a_nb_n$ , then if it be required to draw the projections of a line through the given point which is parallel to the given line, we have only to draw through  $a_1$  the line  $\gamma_1\delta_1$  parallel to  $a_1b_1$  in the horizontal plane, and through  $a_n$  the line  $\gamma_n\delta_n$  parallel to  $a_nb_n$  in the vertical plane. These will be the projections required; and the traces of the line,  $\gamma_1$ ,  $\delta_n$  will be determined as already explained in II.

Having disposed of these preliminary matters (which certainly ought to be amongst the most ordinary topics of professional education, and I trust the day is not distant when they will be so), I proceed to the main purpose of this letter, viz.

V. Fig. 7.—The projection of Architectural Shadows.

I. For Points.—Let  $OX$  be the ground-line,  $R_1$ ,  $R_n$  the horizontal and vertical projections of the pattern-ray,  $a_1$ ,  $a_n$  the projections of a point, and  $b_1$ ,  $b_n$  those of a second point.

Through  $a_1$ , draw the projections of a line parallel to  $R_1$ ,  $R_n$ , and find its traces  $\gamma_1$ ,  $\delta_n$ . Now the visible one of these traces is  $a_1$  on the vertical plane, and is the shadow of the given point, whose projections are  $a_1$ ,  $a_n$ .